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HISTORY AND PHILOSOPHY OF MATHEMATICS

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We consider the Appropriateness of Utilization of Non-Euclidean Mathematics in Relativistic Philosophies

The new era of modern mathematics emerged in the late nineteenth century with the introduction of non-Euclidean geometry. The social and philosophical implications of this mathematical development redirected the Western thought in the way that there would be no return to the Euclidean worldview of absolutes. Non-Euclideanism came to be employed as the scientific rationalization for ethical relativism and its implications. The issue of whether such utilization is appropriate is decisive in tracing or, in this case, renouncing the relationship between non-Euclideanism and ethical relativism. It is undeniable that mathematics has influenced world thinking with respect to ethics and religion for centuries. Yet, the philosophy of relativism can not, as much as it is desired, gain a proponent in the mathematical theories of non-Euclideanism. In fact, much of its implications have been erroneously grounded on various misconceptions of the theories rather than their physical content. Non-Euclidean mathematics does not imply, as is commonly assumed, and should not be associated with the philosophy of ethical relativism, which subsequently undermines faith in the absolute truth. The mathematical concept of relativity does not mean an abandonment of absolute truths; it only means that truth can be formulated in various ways.

For nearly two thousand years, Euclidean geometry had been the foundation and the framework of all mathematics. Not only that, it was hailed as the model of certainty in human knowledge. Euclid's accomplishment was to apply the axiomatic method to geometry, where geometrical principles were reduced to five postulates, from which the rest could be logically deduced. "The simplicity and thoroughness of his systematization lent to geometry an aura of universal and irrefutable truth- [3, p.125]. Of the five axioms of Euclid, the fifth is the one that is less self-evident and seems to transcend direct physical experience. It came to be known as Euclid's Parallel Postulate and is stated as follows:

If a straight line falling on two straight lines makes the interior angles on the same side less than two straight angles, then the two straight lines if extended will meet on that side of the straight line on which the angles are less than two right angles [3, p.141].

The axiom was tried for its validity by attempting to derive it from the other axioms, but the fact remained that it cannot be so derived. The development of

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non-Euclidean geometry was the direct result of these attempts to deal with the Fifth. In their efforts to prove the theorem mathematicians employed the method of reductio ad absurdam in hopes of reaching a logical contradiction at some point. But to their great surprise, the expected contradiction was never reached. The system was found to be consistent in terms other than Euclid's, namely, given in a plane a line l and a point p not on l, (a) there are no lines through p parallel to l, and (b) there is an infinity of lines through p parallel to l. New systems of geometry were being discovered, genuine forms of geometry in the sense that they possessed a valid consistent logical structure. Both geometries were further developed by Bernhard Riemann (1826-1866), the (a) system, and Nikolai Ivanovich Lobachevskii (1793-1856), the (b) system.

Riemannian geometry can be applied when dealing with surfaces of positive curvature, as in a sphere where a 'straight' line is like the arc of a great circle, and, subsequently, the sum of the angles of a triangle is greater than 180^o . Lobachevskian geometry, in contrast, deals with surfaces of negative curvature, and in which the sum of the angles of a triangle is always less than 180^o. Pearcey and Thaxton compare the Riemannian "straight" lines to the longitudinal lines on a globe, and Lobachevskian, to the lines that run lengthwise along a trumpet-like surface [3, p.143]. The discovery of new geometries was met as a disaster in light of the philosophical implications, which it was thought to entail:

These stunning surprises exposed the vulnerability of the one solid foundation — geometrical intuition — on which mathematics had been thought to rest. The loss of certainty in geometry was philosophically intolerable, because it implied the loss of all certainty in human knowledge [1, p.331].

Non-Euclidean geometry was seen as undermining the exemplar of an absolute and "true" knowledge in the face of Euclid's system of geometry, which had always been identified as consistent with and paralleled to the Christian system of ethics. Now it was proven geometrically that one could start with a different set of axioms and from there produce a new consistent system. Among other newly emerged proponents of ethical relativism, French philosopher Jacques Rueff argued that it was in the same manner possible to create non-Euclidean systems of morality [3, p.154]. So it appeared no longer certain that any universal and all-abiding absolute truth existed. In consequence, the existence of God was debated on the grounds of the new developments in mathematics. In this new spirit of relativism, anthropology examined various cultures and "attributed to each [one] its own validity and integrity" [1, p.208]. In this light, Christianity was now perceived in error in its efforts to convert other cultures and adapt the world to its own system of beliefs.

The discovery of non-Euclidean geometries found its completion in Einstein's theory of relativity. The early suspicions that physical universe might be non-Euclidean were manifested in Gauss' effort to test the Euclidean character of terrestrial geometry by triangular measurements from mountain tops in hopes that the sum of the triangle would turn out to be greater than 180^o , as in Riemannian elliptical geometry. The results were inconclusive, but only due to the fact that the difference would not show up until the dimensions are as large as the earth itself. The laws of Euclid described the world of our environment to a high degree of precision; but their suitability to all experiences was mistakenly taken for absolute certainty. Astronomic dimensions require a different kind of framework, where these empirical laws must be abandoned. Eventually, Einstein popularized Riemannian geometry by applying it in astronomical contexts with his concept of curved space [3, p.142]. In a letter to Arnold Sommerfeld in 1915, Einstein acknowledged:

I saw clearly that a satisfactory solution could be found only by means of a connection with the universal theory of covariants of Riemann [5, p.100]...

The theory of relativity's central effect was to make time and space relative. Its logical basis is the discovery that many statements, which were regarded as capable of demonstrable truth or falsity, are mere definitions [4, p.293]. In formulating his hypothesis about the physical world Einstein employs geometrical terminology. Yet, this assumption is not of a geometrical nature, since geometry deals with undefined objects. Einstein's assumption deals with physical objects and amounts to the hypothesis that they behave like points and lines of a non-Euclidean rather than Euclidean space [2, p.464]. His fundamental idea is that a light ray follows the shortest path, and that path has the properties of lines in a non-Euclidean space.

It is now evident that physical space can be described both in Euclidean and non-Euclidean terms, and thus the utilitarian argument against the validity of non-Euclidean geometry becomes obsolete. Still, the philosophical implications of mathematical relativism are thought to point to ethical relativism, and the spirit of uncertainty prevails in theological realms as in many others. Pearcey quotes historian Paul Johnson:

Mistakenly but perhaps inevitably, relativity became confused with relativism... It formed a knife... to help cut society adrift from its traditional moorings in the faith and morals of Judeo-Christian culture [3, p.165].

Einstein himself resisted this misapprehension and the efforts to interpret his theory into a philosophical system of ethical relativism. Indeed, Einstein's theory does not supply evidence for such a sweeping generalization. According to Reichenbach,

The parallelism between the relativity of ethics and that of space and time is nothing more than a superficial analogy, which blurs the essential logical differences between the fields of volition and cognition [4, p.289].

Nor does the theory account for the proposed absence of absolutes in the physical universe. On the contrary, Einstein expressed his strong convictions about the harmony of the universe [4, p.292], which testifies to the existence of some universal principles that govern the creation. Although, in Einstein's view, God was merely a name for the principle of order within the universe [3, p.184], he did acknowledge that the universe could not have been arranged in any other way. Given any of the constants is changed in the slightest of the degrees, the universe would simply cease to exist. This incredible revelation serves as a valid argument for the existence of a supreme mind that governs the universe.

The term "relativity" should be interpreted as meaning "relative to a certain definitional system." Relativity implies plurality of equivalent descriptions, but that plurality is not a plurality of systems of contradictory content. Relativity thus does not mean an abandonment of truth, but rather that it can be formulated in different ways. Different geometries have a place in the physical world, all are consistent within their own terms. To this day we can not account for all the forces that govern the universe, similarly, we can not expect to possess the full knowledge of all the systems in which the creation can be accounted for. All geometries, in this sense, are part of a larger picture, in which all of them fit together, each playing its role.

We can not afford to adhere to a single system of mathematics only because such seems to agree with our immediate spatial experience. The axiomatic mathematical formulation must be separated from this spatial experience. Hopefully in the future we will develop an ability to visualize and regard the laws of non-Euclidean geometry as necessary and self-evident, in the same way as the laws of Euclidean geometry appear to us today.

Of the general theory of relativity you will be convinced, once you have studied it. Therefore I am not going to defend it with a single word (Albert Einstein, February 8, 1916).

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