Математические
структуры и моделирование
1999. Вып. 4, с.89-97.

УДК 578.087:519.6

AN APPROXIMATION FOR LARGE-SCALE BRAIN ACTIVITY

R. Faizullin, V. Melnikov

Finite element method and networking techniques are realized for the simulation of complex behavior of the co-operative brain activity. The electroactivity on the cortex surfaces and other brain structures can be decomposed on the two co-ordinated components. We can say the «charges» - neuron's spikes and about «potential fields» - more slow activity. Saccadic imaging can be transformed to the impulsation along cortex layers, motor reaction can be considered as complex sum of interactioned information. There is tally with the rhythms of the cortex and first eigenvalues of associated operator, there is EEG desynchronization.

1. Analogies

Let us consider neuron's reaction on the contribution of its environment (Schmidt 1983). We can see on the Fig.1 that before $t = 0$ there is intricate behavior for the difference ψ of cross-diaphragms potentials. After time $t = 0$ we have standard spikewise neuron's reaction. Before time $t = t_1$ we can see rapid ψ increasing , and after t_1 we have ψ diminishing with more slowly character than for previuos increasing.

For times between 0 and T the neuron is closed for the coercion of its neighboures, it is so-called- autoregenerativics process. After time T we can see intricate process once again.

Hence, there are an analogies between spikes motion along the layers of neuros tissue and charge's transfering via the gradient of electrodynamical (or electrohydrodynamical) field. There is external contribution, similar to an external field contribution for natural charges, and also forward charges induction. Next spike is to be generated in the closely-spaced neuron which is bound to be neuron with minimal value of the difference between threshold potential – ψ_t and $\psi(t)$. Then we can say about spikes – so it are «charges» and spatial distribution of the $\psi(t)$ defined gradient of some potential function ϕ. For simple linear case we can write:

E-mail: faizulin@univer.omsk.su
Омский государственный университет

$$\rho_t + (\phi_x \rho)_x + (\phi_y \rho)_y = g \tag{1}$$

where $\rho(x, y, t)$ – is surface density of the charges on the layer, and g – the right part is equal to the sources from the other layers. Remark that it is only one of the possible approaches, so we can consider an charges without density, as the sources.

If we know $\rho(x, y, t)$ in arbitary moment of the time then we know ψ, ψ_t for active neurons and for others. Hence, for small step τ or in the moment equal to $t + \tau$ system behavior can be described only with the distribution of $\psi(t)$, but not ψ_t. Then we can write Shrödinger's equation for ϕ:

$$i\phi_t = L(\phi) + f \tag{2}$$

where L – is a linear operator and f describes a sources from other layers. We can consider L as elliptic, second order differential operator with arbitary (may be only first) coefficients. For example, it can be choosen as Laplace operator.

We must describe cross-layers induction in these coefficients and in coefficients by (1). Also, there is analogy with electrohydrodynamical problem (Poljanski 1994).

There is more complex nonlinear behavior during long time for charges and stream function's lines when we have some edges on the blades We hope that the behavior of the activity on the layers of brains structures which modelling as dependent variables will be intricate too.

2. Simulation for brains structures

The equations under consideration can be connected with the «layers» which can be associated to real layers of neuros tissue. Let us consider approximative mesh which lies similar to one of cortex layers (like for current structure analysis problems). Geometrical propeties of the structures and their connections (Fig. 2) can be defined by some well known finite element codes.

For example, crude mesh for cortex model can be constructed of the form of two connected hemisheres which are cuted by the line along motor zone, Fig.3.

On the one side of this line we have Dirichlet condition, and we can consider this as motor zone, but on the other side we have nonhomogeneous Neumann condition for the modelling of somatosensor's information flow in proper direction.

At the first, external field must be defined from Laplace problem solution, without the charges but with potential difference on primary and motor zone for later flow of the charges. Visual information can be simulated from the distribution of unsteady sources on the board of visual zone of the cortex like to charge-producings on metalic boundaries for electrohydrodynamics systems. For example, one can construct external method based on *saccadic* movements of the eyes. At the first, charges on the sensory zone of the cortex were simulated as the quasi-Fourier coefficients obtained by specially designed pattern recognition codes. Another approach deals with direct projection of a external picture to triangulational mesh for the imitation of the result of activity for the primary cortex.

Initially, we have considered the charge as a the continuous function. But from the experemental physiological data, we know that the part of active neurons comparatively not too large, and we know experimantal facts about the localization of cortex activity. Therefore we can model the spreading of spikes better than earlier, with a discrete charge which extends by the field simulated via potential. But we must consider that ϕ is not same every time, when the spikes affects to an underboundary potential.

The charges or in other words the spikes are going from the zones to motor line. Toroidal mesh is included to every hemishere and models brains limbic system. The charges are going from cortex to limbic system and from limba to prefrontal zone of the cortex. Thalamic system and the sum of all other brain's parts can be simulated like an parabaloid which is united with prefrontal lob zone and with the parts of limbic model. We can try to change variable coefficients in (1) , (2) to simulation the memory for current flows.

Finite element method and networking techniques are realized for global simulation which include brain imaging and activity interactions (Fix 1977).

By the way, we can observe not only general picture of the spike's distribution, but also pure field induced by individual charge and its trajectory. The observation of the interactions between some charges, may be only two, can give us the result of valid deviation of the particle's way from initial charge's trajectory- without other charges.

3. Eigenvalue problem and EEG

Let us consider eigenvalue problem for Laplace operator which can be defined on the cortex zone:

$$-\Delta v_i = \lambda_i v_i, \tag{3}$$

where λ_i, v_i is i th pair of the eigenvalue and the eigenfunction of Laplace operator. Lanczos algorithm was used to solve this problem like for well known mechanical problems (Parlett 1980).

There is unexpected tally with rhythms of cortex ($\theta, \delta, \alpha, \beta_1, \beta$) for first eigenvalues of associated operator but only in the cases when we take quasi-one-dimensional problem. We consider Dirichlet condition on motor zone and Neumann's condition on sensor zones. So, first eigenvalue is by two order smaller than second eigenvalue, which can be associated with θ rhythm for the brains rhythm (Schmidt 1983) . Third eigenvalue associated with δ rhythm and others with α, β_1, β rhythms. Their fractions approximate rhythms fractions $3:6:10:15:25$ in the average.

Then we tried to take encephalographic line, like in medical practice. We registrated potential difference between two points of one hemisphere. We take a values of potential at 10 meshpoints on two hemispheres (5 from right and 5 from left). Thus from every hemisphere we have gotten 10 encephalographic lines. Every of those is like oscillating process of potential difference near zero, moreover the amplitude of the oscillations does not exceeds a definite boundary. When we obtained correspondence between our datum and the datum from medical practice, we could consider more subtle structure phenomena – α rhythms desynchronization.

Then, when the number of the neighbours decreased at the instant the charges leaves node of the mesh, we can see positive shift of oscillating frequencies of encephalographic function. More exactly, there is amplitude increasing for the frequencies higher order (Melnikov 1996). It's seems as simulation of the EEG desynchronization phenomena on the Fig. 4, 5.

4. An attempt to model the external environment

Then we made an attempt to model the external environment and the answer to it's influence.We have y as a deformation of an organism. The charges on the motor zone we can consider as same stepped function.We have a equation for y and f. Let

$$-y_{\phi\phi} = f(\phi) \tag{4}$$

$$y(0) = y(2\pi), \tag{5}$$

where f expands into Fourier series of the charges function without 1st coefficient and ϕ is an angle. When the organism is relaxed (we have no charges on motor zone),it is like a simple circle. But, when the charges came , the surface of it lose shape. On the circle we have 3 points that defines 3 vectors, the sum of those vectors give to us the direction and quantity of movement of our «animal». This organism moves on a field with pictures and reacts to them as the pictures projects on the part of the brains mesh.

5. Network realization

Sumulations was running on the poor(386s) Netware net.

First of the net's computers worked under simulation of 'virtual world'- creature's motion in the 2-D space (box with some objects).

Second net's computers worked under visual and somatosensor imaging.

Third net's computer worked under creation of motor reaction- general motion plus creature's boundary deformation.

Every part of the brain was considered as a 'layer'. For every 'layer' was related one of the next net's computers where was calculated charges motion, potential media coefficients changing on same layer. There was sited information of layer's geometry plus on-line information of charges and potentials.

Net's server was considered as a place for temporal files- values of charges and potentials on layers, on-line information of virtual world, values of charges on motor zone and on the boundaries of sensor's zones. Files was downloaded (from layer) and uploaded (from neighbouring layers) with uncoherent delays (to prevent collisions).

Our aim was not optimization but searching answers on the next questions- can our creature 'live' during at a long time, are there global effects similar for a real effects (EEG and rhythms) ? Now we try include IP connections in our model plus direct file transfer between layers (simulation without main server).

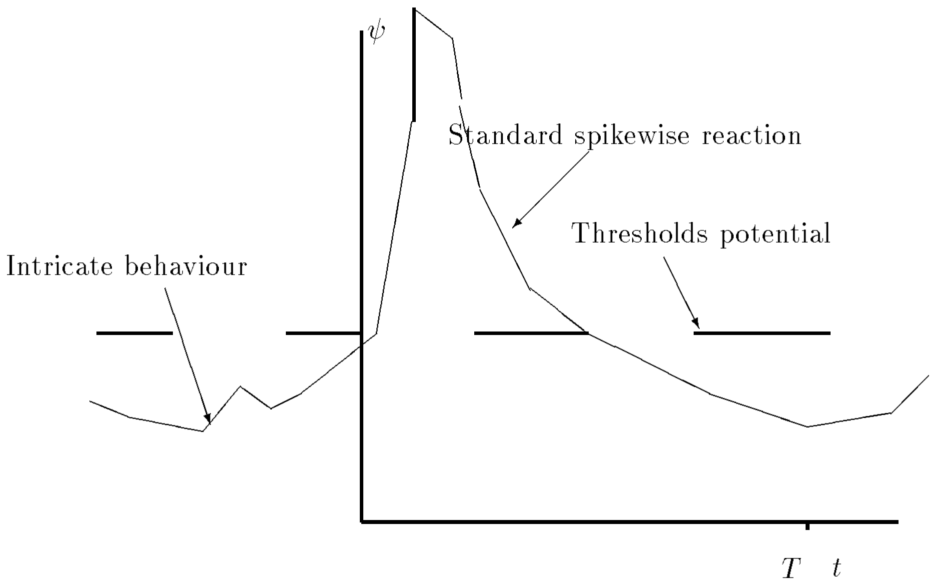

Fig.1. Difference of the diaphragm's potential $-\psi$,
spike generation, from 0 to the T
neuron is closed for external stimulation

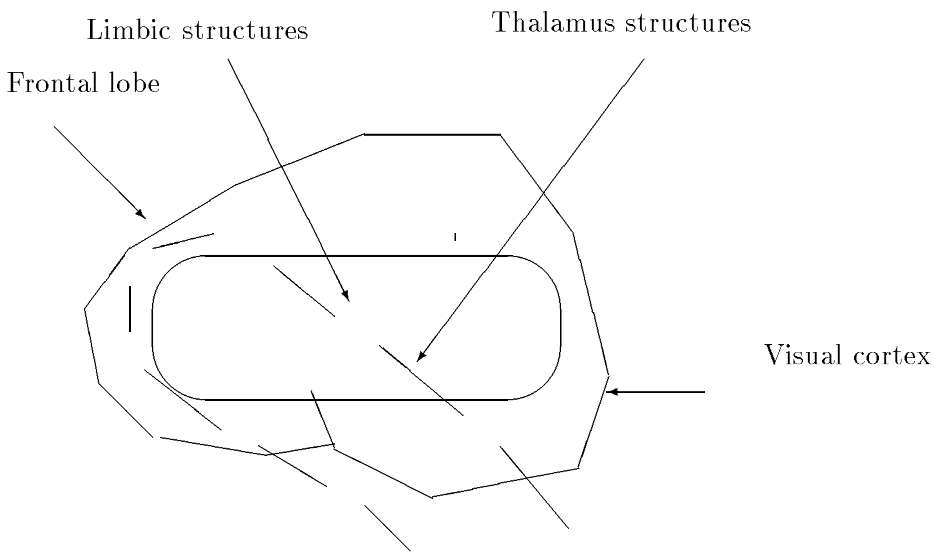

Fig.2. Approximation of the brains structures,
view from the side, cortex,limba,thalamus

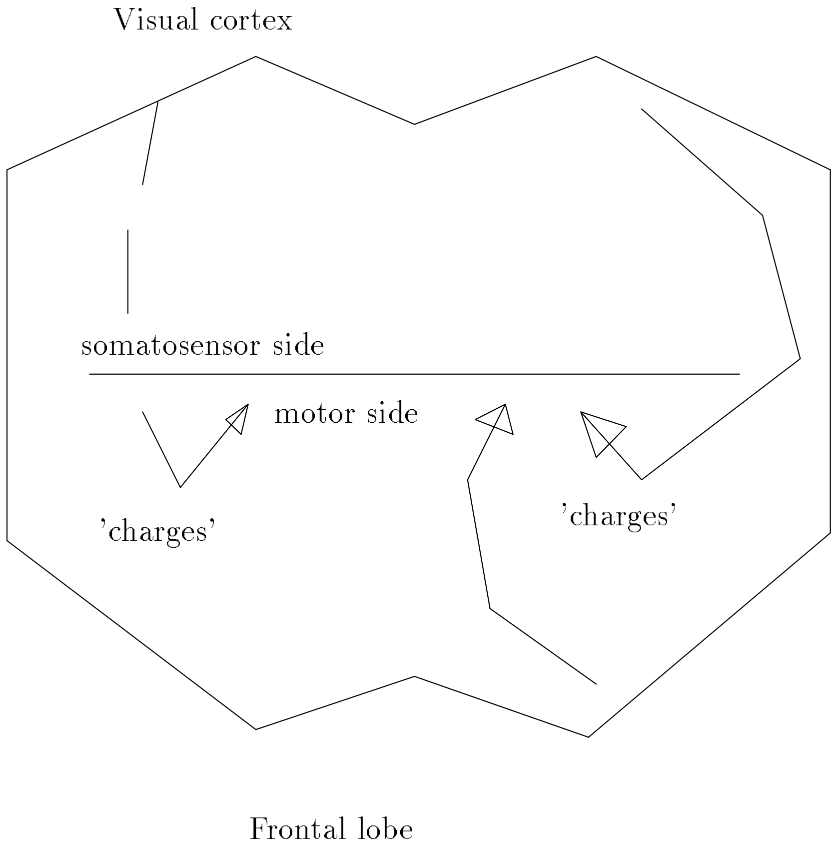

Fig.3. Charges, top view

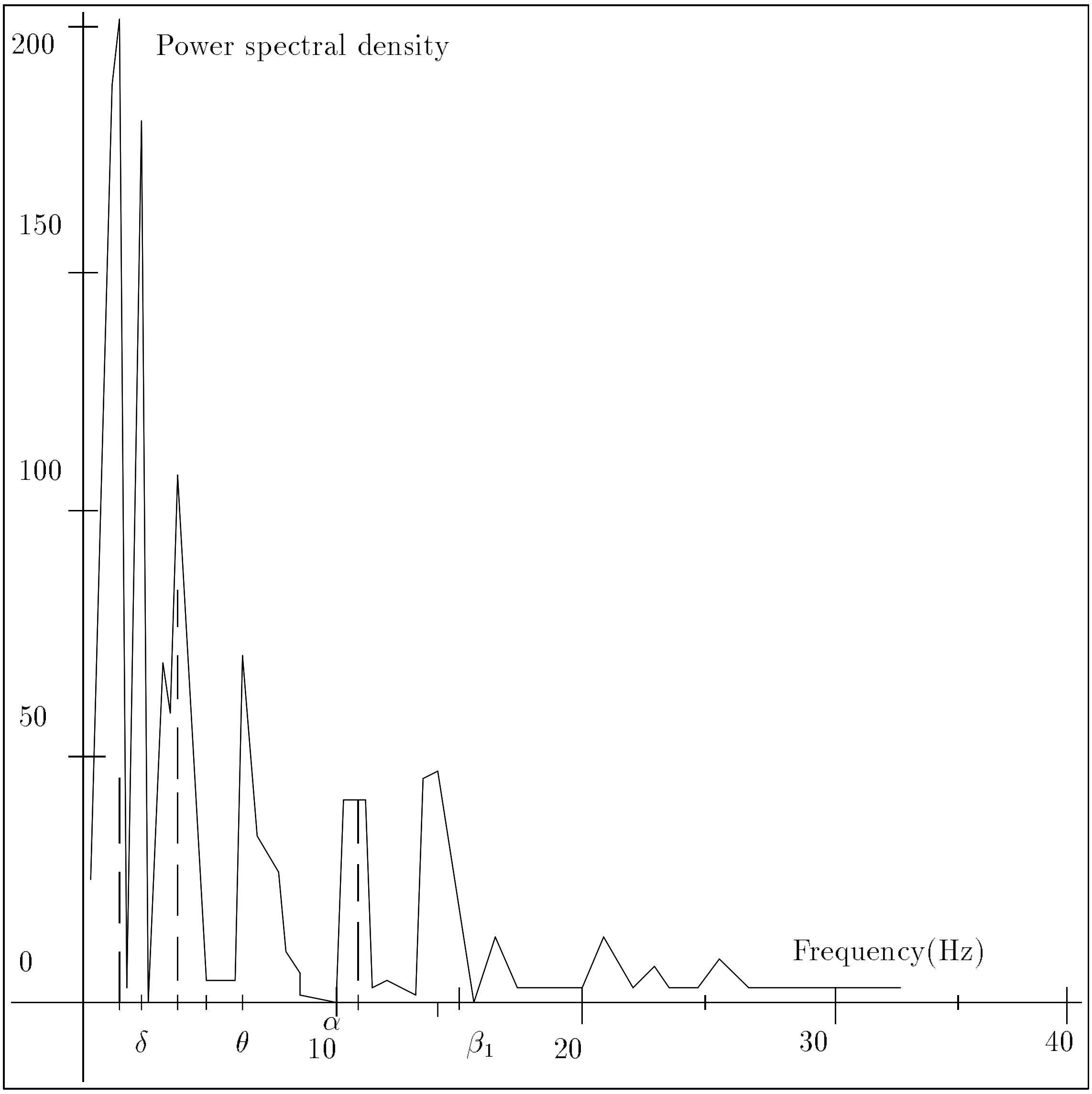

Fig.4. Power density for the standard numbers of the mesh relations

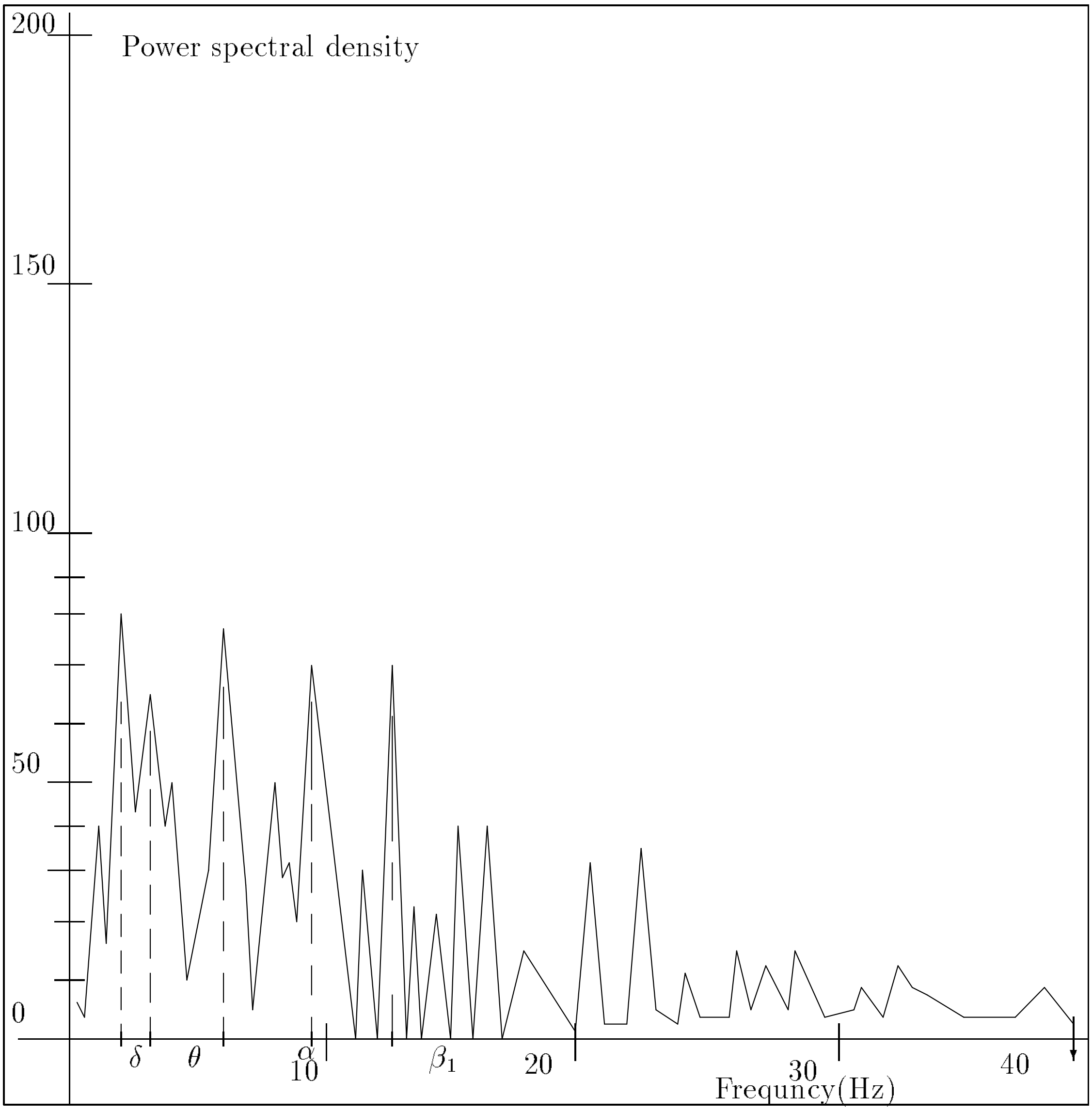

Fig.5. Power density for the decreasing numbers of the mesh relations

Литература

1. Fix G.F. and Strang G. *Theory of finite element methods.* Moscow: Mir, 1997.

2. Melnikov V.V. *The Unsteady Simulation of Brain's Patterns.* Diploma work, Dept. of Math. Omsk State University. Omsk. Russia. 1996.

3. Parlett B.N. *The Symmetric Eigenvalue problem* // Prentice-Hall, Inc., Englewood Cliffs. 1980.

4. Poljanski V.A., Sakharov V.I., Faizullin R.T. *Unsteady hydroelectrodynamics calculations in pumps* // Proceedingsof the 3 Int.Symp. Hydroelecrtodynamic St.Peterburg. Russia. June. (St Pet Univ.). P.134-137 (in Russian).

5. Schmidt R.F. and Thews G. *Human physiology* // Springer-Verlag. N.Y. 1983.